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$$\therefore s=-\tfrac{1}{2}\left[\frac{a^{\beta}\,(a-x)^{1-\beta}}{1-\beta}+\frac{a^{-\beta}\,(a-x)^{1+\beta}}{1+\beta}\right]+C_3.\quad \text{For } x=0,\ s=0.$$

$$\therefore C_3=\frac{a}{1-\beta^2}.\quad \therefore s=\frac{a}{1-\beta^2}-\tfrac{1}{2}\left[\frac{a^{\beta}\,(a-x)^{1-\beta}}{1-\beta}+\frac{a^{-\beta}\,(a-x)^{1+\beta}}{1+\beta}\right].$$

Since for $x=a$, $y=b$, we have $b=\dfrac{a^{\beta}}{1-\beta^2}$; or, substituting $a=100$, $b=300$, we have $\beta^2+\frac{1}{3}\beta=1$; whence $\beta=\frac{1}{6}(\sqrt{37}-1)$ and length of curve between A and $C=\dfrac{a}{1-\beta^2}=50(\sqrt{37}+1)=354.135$.

$$y=\frac{na}{n^2-1}-\frac{na^{1/n}}{2(n-1)}(a-x)^{(n-1)/n}+\frac{n}{2(n+1)}\frac{1}{a^n}(a-x)^{(n-1)/n}.$$

315. Proposed by C. N. SCHMALL, New York City.

If $y=f(x)$, show by Taylor's Theorem that

$$f\left(\frac{x}{1+x}\right)=y-\frac{x^2}{1+x}\cdot\frac{dy}{dx}+\frac{x^4}{2\,(1+x)^2}\cdot\frac{d^2y}{dx^2}-\frac{x^6}{2.3.(1+x)^3}\cdot\frac{d^3y}{dx^3}+\ldots \text{ etc.}$$

Solution by the PROPOSER.

Put $\dfrac{x}{1+x}=x+h$, then $f\left(\dfrac{x}{1+x}\right)=f(x+h)$,

$$\text{and also, } h=\frac{x}{1+x}-x=-\frac{x^2}{1+x}.$$

$$\therefore h^2=\frac{x^4}{(1+x)^2},\quad h^3=-\frac{x^6}{(1+x)^3},\ \text{and so on.}$$

Substituting these values of the powers of h in Taylor's series, we have the required result; *i. e.*, from

$$f(x+h)=f(x)+hf'(x)+\frac{h^2}{2!}f''(x)+\frac{h^3}{3!}f'''(x)+\text{etc.},$$

we get, by direct substitution,

$$f\left(\frac{x}{1+x}\right)=y-\frac{x^2}{1+x}\frac{dy}{dx}+\frac{x^4}{2\,(1+x)^2}\frac{d^2y}{dx^2}-\frac{x^6}{2.3(1+x)^3}\frac{d^3y}{dx^3}+\text{etc.}$$

316. Proposed by C. N. SCHMALL, New York City.

$$\int_0^{\infty}\frac{\cos ax}{1+x^2}dx=\tfrac{1}{2}\pi e^{-a}=\int_0^{\infty}\frac{x\sin ax}{1+x^2}dx.$$

(From Bromwich, *Theory of Infinite Series*, p. 442, ex. 5, and also from Carslaw, *Fourier's Series*, p. 113, ex. 12.) Prove this by any method.

Solution by A. M. HARDING, Fayetteville, Arkansas.

It can easily be shown that

$$\int_0^\infty e^{-bx}\cos bm\,db=\frac{x}{m^2+x^2};\quad \int_0^\infty e^{-bx}\sin bm\,db=\frac{m}{x^2+m^2}$$

$$\text{and } 2\int_0^\infty be^{-b^2(1+x^2)}db=\frac{1}{1+x^2}.$$

$$\text{Hence } u=\int_0^\infty \frac{x\sin ax}{1+x^2}dx=\int_0^\infty \sin ax\left(\int_0^\infty e^{-bx}\cos b\,db\right)dx.$$

$$\therefore \frac{du}{db}=\int_0^\infty \sin ax\,e^{-bx}\cos b\,dx=\cos b\int_0^\infty e^{-bx}\sin ax\,dx=\cos b.\frac{a}{a^2+b^2}.$$

$$\therefore u=\int_0^\infty \frac{a\cos b}{a^2+b^2}db,\text{ putting } ax \text{ for } b,=\int_0^\infty \frac{\cos ax}{1+x^2}dx.$$

$$\therefore \int_0^\infty \frac{x\sin ax}{1+x^2}dx=\int_0^\infty \frac{\cos ax}{1+x^2}dx.$$

$$\text{Now } u=\int_0^\infty \frac{\cos ax}{1+x^2}dx=\int_0^\infty \cos ax\left(2\int_0^\infty be^{-b^2(1+x^2)}db\right)dx.$$

$$\frac{du}{db}=2\int_0^\infty \cos ax.be^{-b^2(1+x^2)}dx=2\,be^{-b^2}\int_0^\infty e^{-b^2x^2}\cos ax\,dx=2be^{-b^2}.\frac{\sqrt{\pi}}{2b}e^{-(a^2/4b^2)}$$

(see Byerly's *Integral Calculus*, Art. 93 (b))

$$=\sqrt{(\pi)}\,e^{-b^2-[(a/2)^2/b^2]}.\qquad u=\sqrt{(\pi)}\int_0^\infty e^{-b^2-[(a/2)^2/b^2]}db=\sqrt{(\pi)}.e^{-a}.\frac{\sqrt{\pi}}{2}=\frac{\pi}{2}e^{-a}.$$

(Art. 93 (a)).

Also solved by J. Scheffer, S. A. Corey, and the Proposer.

NOTES AND NEWS.

Professor Cajori's *Theory of Equations*, which was published in 1904, was reprinted by the Macmillan Company during last January. M.

The spring meeting of the Chicago section of the American Mathematical Society will be held at Chicago on Friday and Saturday, April 5 and 6, 1912. S.